Dynamical analysis of the soil-foundation interaction system due to lateral excitations

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ABSTRACT: In order to obtain a theoretical prediction on the seismic response of the soil-foundation interaction systems, the frequency response of the soil-foundation system excited by ground incident waves may be necessary in addition to the response due to the excitation at the head of foundation. This paper is concerned with a theoretical analysis based on the three-dimensional wave propagation theory to find dynamical characteristics of an elastic foundation embedded in the viscoelastic soil stratum on a rigid bed rock, when laterally subjected to a concentrated external excitation on the head of the foundation and uniformly distributed bed rock motion. In dealing with this complicated boundary configuration and exciting condition, the technique of superposition principle associated with the auxiliary subproblems and the integral transform method with respect to the spatial variables are effectively used. And, the governing equations in frequency domain reduce to the infinitely simultaneous equations between the coefficient functions corresponding to the subproblems.

INTRODUCTION

There has recently been an increasing interest in the dynamic characteristics of a foundation-ground interaction system in the field of structural engineering. General approaches to such a dynamic interaction problem based on the three-dimensional wave propagation theory are to be related to solve a class of mixed boundary value problems with complex boundary configurations.

The objective of this paper is to present a method of theoretical analysis of a foundation-ground interaction system consisting of an elastic circular cylindrical foundation and the surrounding linear viscoelastic semi-infinite soil medium when laterally or rotationally subjected to the concentrated external excitation at the head of foundation and to uniformly distributed bed rock motion.

In dealing with these complicated boundary value problems, the total soil-foundation interaction field is separated into the incident-field motion corresponding to the external excitations and the interacted-field due to the presence of the intermediate boundary between a foundation and soil-stratum. The latter field is further separated into the two sets of fields, corresponding to the following auxiliary subproblems;

(z) one related to the cylindrical boundaries parallel to the symmetrical axis of a foundation, and

(r) the other related to the plane boundaries parpendicular to that axis of a foundation. Then, the respective stress and displacement components of the auxiliary problems are combined to satisfy the original boundary conditions. By applying the Fourier transform with respect to time to the wave equation describing the vector displacement field, and by introducing scalar potentials associated with the cylindrical polar coordinate system, the governing wave equations reduce to scalar Helmholtz type wave equations. The Fourier and Hankel integral representations with respect to spatial variables are applied to obtain the governing equations in the domain of wave number, and in consequence, the Fredholm type simultaneous series equations of the third kind for the unknown coefficient functions of the potentials are derived through matching the boundary conditions. In solving the above equations numerically, the results for displacement, force and stress field in frequency domain are presented for some physical properties of a soil-foundation interaction system.

FORMULATION OF THE PROBLEM

The displacement vector u of the interaction system composed of the cylindrical foundation (I) and its surrounding soil stratum (II) is required to satisfy the following mixed boundary value forms,

$$
\begin{aligned}
&L_\nu(u_\nu)=0 && x \in V_\nu \\
&\beta_{\nu 0}(u_\nu)=\gamma_{\nu 0} && x \in \Gamma_{\nu 0} \qquad (1)\\
&\beta_{\nu 1}(u_\nu)=\gamma_{\nu 1} && x \in \Gamma_{\nu 1} \\
&\beta_{I\ II}(u_I,u_{II})=0 && x \in \Gamma_{I\ II} \\
& && :\ \nu=I,II
\end{aligned}
$$

in which u_ν, x are the displacement vector and the position vector in the medium V_ν and,

(i) the three-dimensional wave equation of the medium given by the vector differential operator L_ν,

(ii) the stress-condition associated with the operator $\beta_{\nu 0}$ at the surface $\Gamma_{\nu 0}$,

(iii) the welded contact displacement-condition associated with the operator $\beta_{\nu 1}$ at the interface $\Gamma_{\nu 1}$ of the medium (ν) and the rigid bed rock,

(iv) the continuous condition between the foundation (I) and its surrounding soil stratum (II) at the interface $\Gamma_{I\ II}$ associated with the operator $\beta_{I\ II}$.

In addition, the radiation condition in the infinitely far field of the medium (II) is required to be satisfied.

It is convenient to write the interaction field in the absolute coordinate system as follows;

$$u_\nu = u_\nu^i + u_\nu^s \qquad :\ \nu=I,II \qquad (2)$$

where u_ν^i is the incident-field motion of the medium (ν) in the absence of another one and u_ν^s corresponds to the interacted-field motion due to the difference between the absolute-field and the incident-field. The free-field motion u_ν^i is required to satisfy the conditions;

$$
\begin{aligned}
&L_\nu(u_\nu^i)=0 && x \in V_\nu \\
&\beta_{\nu 0}(u_\nu^i)=\gamma_{\nu 0} && x \in \Gamma_{\nu 0} \qquad (3)\\
&\beta_{\nu 1}(u_\nu^i)=\gamma_{\nu 1} && x \in \Gamma_{\nu 1} \\
& && :\ \nu=I,II
\end{aligned}
$$

in which the displacement field u_ν^i is obtained in the closed forms, as mentioned afterwards.

Therefore the equations requested for

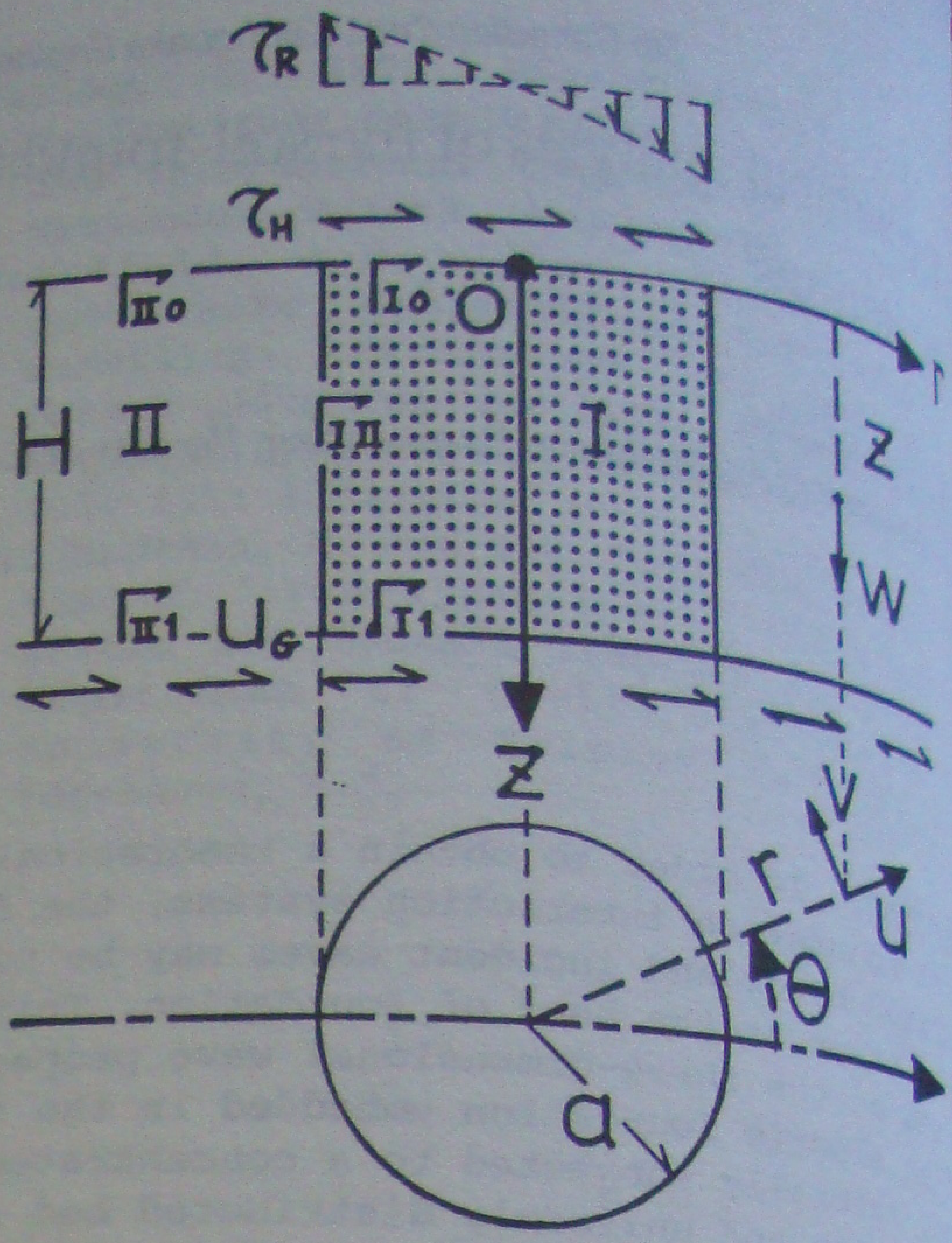

Fig.1 System configuration and coordinate system

the interacted-field u_ν^s have the similar form as in the case of the absolute-field in absence of the external excitation, namely Eq.(1), but the inhomogeneous term f in this case is determined by using the incident-field motion.

$$
\begin{aligned}
&L_\nu(u_\nu^s)=0 && x \in V_\nu \\
&\beta_{\nu 0}(u_\nu^s)=0 && x \in \Gamma_{\nu 0} \\
&\beta_{\nu 1}(u_\nu^s)=0 && x \in \Gamma_{\nu 1} \qquad (4)\\
&\beta_{I\ II}(u_I^s,u_{II}^s)=f && x \in \Gamma_{I\ II} \\
& && :\ \nu=I,II
\end{aligned}
$$

The interacted displacement vector field u_ν^s presented in cylindrical polar coordinates (r,θ,z), as shown in Figure 1, in which the symmetrical axis of the foundation coincides with the z-direction, can be expressed in terms of potentials of dilatational and distortional components as follows;

$$u^s = \nabla\varphi + \nabla\times(\psi e) + \nabla\times\nabla\times(\chi e) \qquad (5)$$

where ∇ and e denote the gradient operator and the unit base vector along the z-axis, φ, ψ and χ are particular solutions of the associated scalar Helmholtz equations;

$$\begin{bmatrix}(\nabla^2+j^2)\ \varphi\\(\nabla^2+k^2)\ \psi\\(\nabla^2+k^2)\ \chi\end{bmatrix}=0 \qquad (6)$$

in which ω is the circular frequency of the harmonic excitation, c_p and c_s are the phase velocities of dilatational and distortional waves, and their associated parameters are $j=\omega/c_p$, $k=\omega/c_s$.

For this interacted-field, the finite Fourier transform is operated with respect to the circumferential direction θ, where only the terms of the operator order 1 are standing remained so as to have the same operatored type of the incident vector field $u^i=(u,v,w)^i$ presented in cylindrical polar coordinate system, by considering the periodicity condition,

$$\begin{bmatrix}u\\v\\w\end{bmatrix}^i=\begin{bmatrix}\cos\theta&&\\&\sin\theta&\\&&\cos\theta\end{bmatrix}\begin{bmatrix}U\\-U\\rW\end{bmatrix}^i \qquad (7)$$

$$\begin{bmatrix}\varphi\\\psi\\\chi\end{bmatrix}=\begin{bmatrix}\cos\theta&&\\&\sin\theta&\\&&\cos\theta\end{bmatrix}\begin{bmatrix}\Phi\\\Psi\\X\end{bmatrix} \qquad (8)$$

where the horizontal translation is subjected to the excitation along the axis of $\theta=0$ on the head of foundation or the surface of the rigid bed rock, and the rotational motion is subjected to the rotating force about the horizontal axis of $\theta=\pi/2$ on the head of foundation.

And, the transformed interacted-field is separated into the two sets of auxiliary displacement fields $U^{(z)}$ and $U^{(r)}$, which correspond to the subproblems mentioned previously,

$$U^s=U^{(z)}+U^{(r)} \qquad (9)$$

given in the following transformed potential forms;

(1) for the subproblem (z) associated with the horizontal translation,

$$\begin{bmatrix}\Phi\\\Psi\\X\end{bmatrix}^{(z)}=\sum_n\begin{bmatrix}\cos p_nz&&\\&\cos p_nz&\\&&\sin p_nz\end{bmatrix}\begin{bmatrix}L_1(\alpha_nr)&&\\&L_1(\beta_nr)&\\&&L_1(\beta_nr)\end{bmatrix}\begin{bmatrix}A\\B\\C\end{bmatrix}_n \qquad (10\text{-}1)$$

(2) for the subproblem (z) associated with the rotation about a horizontal axis,

$$\begin{bmatrix}\Phi\\\Psi\\X\end{bmatrix}^{(z)}=\sum_n\begin{bmatrix}\sin p_nz&&\\&\sin p_nz&\\&&\cos p_nz\end{bmatrix}\begin{bmatrix}L_1(\alpha_nr)&&\\&L_1(\beta_nr)&\\&&L_1(\beta_nr)\end{bmatrix}\begin{bmatrix}A\\B\\C\end{bmatrix}_n \qquad (10\text{-}2)$$

(3) for the subproblem (r) corresponding to the contributions of the horizontal plane surfaces,

$$\begin{bmatrix}\Phi\\\Psi\\X\end{bmatrix}^{(r)}=\int d\zeta J_1(\zeta r)\begin{bmatrix}D_1\cosh\alpha z+D_2\sinh\alpha z\\E_1\cosh\beta z+E_2\sinh\beta z\\F_1\cosh\beta z+F_2\sinh\beta z\end{bmatrix}_\zeta \qquad (10\text{-}3)$$

where p_n and ζ are the discrete and continuous parameters of wave number, and their associated parameters are;

$$p_n=(2n-1)/2H,\quad n=1,2,3\cdots\cdot$$
$$\alpha_n=\sqrt{p_n^2-j^2},\quad \beta_n=\sqrt{p_n^2-k^2},$$
$$\alpha=\sqrt{\zeta^2-j^2},\quad \beta=\sqrt{\zeta^2-k^2}.$$

and $J_1(x)$, $L_1(x)$ are the Bessel and modified Bessel functions of the integer order 1, by letting the latter function correspond to $I_1(x)$ for the medium (I) or $K_1(x)$ for the medium (II).

The stress and displacement components derived from the potentials of the subproblems (z) and (r) are superposed to satisfy the boundary conditions requested for the interacted-fields, namely Eq.(4). In order to derive the boundary equations in the domain of wave number for the plane surfaces parallel to the radial direction, the Hankel transform with respect to r is applied to the potentials of the auxiliary problem (z). They are multiplied by the cutoff operator $1-U(r-a)$ for the medium (I), or $U(r-a)$ for the medium (II) to avoid the singularity of the modified Bessel functions, where $U(x)$ means the unit step function and a is the radius of the cylindrical foundation, and by expressing any component on the surfaces in terms of the Hankel integral representation, the resulting integrand except for the Bessel function is concerned with the surface Rayleigh waves travelling along the horizontal axis.

Similarly, for the cylindrical boundary surface along the symmetrical axis of foundation, the finite Fourier transform with respect to z is applied to the potentials of the subproblem (r) and the inhomogeneous terms f.

In consequence, all terms of each boundary equation are arranged to have the same transform operator, and any spatial variable disappears, so that the mixed equations composed of the Fredholm type simultaneous series equations of the third kind determining the unknown coefficients of the potentials $(z^I\ z^{II}\ z^M)_n$ are obtained in the form,

$$\begin{bmatrix} A^{I1} & A^{I2} \\ A^{II1} & A^{II2} \\ A^{M1} & A^{M2} \end{bmatrix}_n^T \begin{bmatrix} z^I \\ z^{II} \\ z^M \end{bmatrix}_n + \sum_m \int d\zeta \begin{bmatrix} B^{I1} & B^{I2} \\ B^{II1} & B^{II2} \\ B^{M1} & B^{M2} \end{bmatrix}_{n\zeta}^T \begin{bmatrix} C^I & & \\ & C^{II} & \\ & & C^M \end{bmatrix}_{m\zeta} \begin{bmatrix} z^I \\ z^{II} \\ z^M \end{bmatrix}_m = \begin{bmatrix} f^1 \\ f^2 \end{bmatrix}_n \quad (11)$$

in which the kernel functions have the integral representations containing the previously mentioned Rayleigh functions, and the unknown functions are;

$$(z^I \ z^{II} \ z^M)_n = \begin{bmatrix} A^I & A^{II} & B^I \\ C^I & C^{II} & B^{II} \end{bmatrix}_n$$

When solving the above series equations numerically, any frequency response of this interaction system is expressed in terms of the unknown functions Y_n^{ν} as follows;

$$u(\omega) = u^i(\omega) + \sum_n a_n(\omega) Y_n(\omega) + \int d\zeta \sum_n b_{\zeta n}(\omega) Y_n(\omega) \quad (12)$$

where

$$Y_n^{\nu} = \begin{bmatrix} A \\ B \\ C \end{bmatrix}_n^{\nu}$$

The first term in the right hand side of this equation shows the contributions of the incident-field, the second term represents mainly the interaction effect of the soil-foundation system in the case of the subproblem (z) and the term remained may correspond to the radiating characteristics of the subproblem (r) for the surface waves propagating to the radial direction.

TRANSLATIONAL EXCITATION ALONG A HORIZONTAL AXIS

For the case of the horizontal translation subjected to the concentrated uniform force on the head of foundation, the incident-field of the medium (I) expanded in the circumferential direction as shown in Eq.(7) is obtained in the following closed and one-dimensional forms;

$$\begin{bmatrix} U \\ W \end{bmatrix}_I^{i(H)} = \frac{\tau_H}{\mu k \cos kH} \begin{bmatrix} \sin k(H-z) \\ 0 \end{bmatrix}_I \quad (13\text{-}1)$$

and for the medium (II), any component of the incident-field is evaluated to be zero. Similarily, for the case of the horizontal translation subjected to the uniformly distributed bed rock motion, the incident-field of the medium (ν=I,II) expanded in the θ-direction is given in the following closed and one-dimensional equations;

$$\begin{bmatrix} U \\ W \end{bmatrix}_{\nu}^{i(G)} = \frac{u_G}{\cos kH} \begin{bmatrix} \cos kz \\ 0 \end{bmatrix}_{\nu} \quad (13\text{-}2)$$

In the fields mentioned here, τ_H and u_G are the amplitudes of the horizontal exciting force on the head of foundation Γ_{IO} and that of the uniformly distributed displacement at the surface of the bed rock $\Gamma_{\nu 1}$, and H, μ_{ν} are the thickness and the shearing constant of the medium (ν).

The interacted-fields of the horizontal translation subjected to the concentrated force on the head of foundation and the uniformly distributed bed rock motion have the same potential forms given by Eqs.(10-1) and (10-3), so that the dominant equations requested for these interacted-fields are different only in the inhomogeneous terms determined by using their incident-field motions mentioned above. And for the boundary conditions on the horizontal surfaces $\Gamma_{\nu o}$ and $\Gamma_{\nu 1}$ demanding the unknown coefficients of the potential term $\Psi^{(r)}$ to vanish, the components $(B^{M1} \ B^{M2})_{n\zeta}$ and $C^M_{m\zeta}$ of the kernel functions in the simultaneous equations (11) are evaluated to be zeros. Therefore, the unknown functions z_n^M can be found to depend upon the other unknown ones in the simultaneous equations, which are exchanged for the more degenerated forms.

For the brevity of expressions, the following dimensionless parameters and dimensionless components with superscript ($\bar{\ }$) are introduced, though the letter will be suppressed throughout the analysis, unless otherwise noted;

$$\begin{bmatrix} \bar{z} \\ \bar{a} \end{bmatrix} = H^{-1} \begin{bmatrix} z \\ a \end{bmatrix}, \quad \bar{r} = a^{-1} r, \quad \bar{k}_0 = \mathrm{Re}(\bar{k}_{II})$$

$$\begin{bmatrix} \bar{p}_n \\ \bar{\zeta} \end{bmatrix} = H \begin{bmatrix} p_n \\ \zeta \end{bmatrix}, \quad \begin{bmatrix} \bar{j} & \bar{\alpha}_n & \bar{\alpha} \\ \bar{k} & \bar{\beta}_n & \bar{\beta} \end{bmatrix}^{\nu} = H \begin{bmatrix} j & \alpha_n & \alpha \\ k & \beta_n & \beta \end{bmatrix}^{\nu}$$

$$\begin{bmatrix} \bar{A}_n \\ \bar{B}_n \\ \bar{C}_n \\ \bar{D}_{\zeta} \\ \bar{F}_{\zeta} \end{bmatrix}^{\nu} = \frac{\mu_0}{\tau_H H^2} \begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & H^{-1} & & \\ & & & 1 & \\ & & & & H^{-1} \end{bmatrix} \begin{bmatrix} A_n \\ B_n \\ C_n \\ D_{\zeta} \\ F_{\zeta} \end{bmatrix}^{\nu}$$

$$\begin{bmatrix}\bar{A}_n\\ \bar{B}_n\\ \bar{C}_n\\ \bar{D}_\zeta\\ \bar{F}_\zeta\end{bmatrix}^{\nu} = \frac{1}{u_G H}\begin{bmatrix}1 & & & & \\ & 1 & & & \\ & & H^{-1} & & \\ & & & 1 & \\ & & & & H^{-1}\end{bmatrix}\begin{bmatrix}A_n\\ B_n\\ C_n\\ D_\zeta\\ F_\zeta\end{bmatrix}^{\nu}$$

$$\begin{bmatrix}\bar{\rho}\\ \bar{\mu}\end{bmatrix}^{\nu} = \begin{bmatrix}\rho_0^{-1} & \\ & \mu_0^{-1}\end{bmatrix}\begin{bmatrix}\rho\\ \mu\end{bmatrix}^{\nu}, \quad \begin{bmatrix}\rho_0\\ \mu_0\end{bmatrix} = \begin{bmatrix}\rho\\ \mathrm{Re}(\mu)\end{bmatrix}^{II}$$

As for the dimensionless displacement field subjected to the horizontal excitations,

$$\begin{bmatrix}\bar{u}\\ \bar{\vartheta}\end{bmatrix}^{(H)} = \frac{\mu_0}{\tau_H}\begin{bmatrix}H^{-1} & \\ & 1\end{bmatrix}\begin{bmatrix}u\\ \partial w/\partial r\end{bmatrix}^{(H)} \qquad (14\text{-}1)$$

$$\begin{bmatrix}\bar{u}\\ \bar{\vartheta}\end{bmatrix}^{(G)} = \frac{1}{u_G}\begin{bmatrix}1 & \\ & H\end{bmatrix}\begin{bmatrix}u\\ \partial w/\partial r\end{bmatrix}^{(G)} \qquad (14\text{-}2)$$

where ρ_0 and μ_0 are the density and the real part of the shearing constant of the soil medium, respectively. In evaluating practically the basic characteristics of the soil-foundation systems, it is assumed that the foundation and its surrounding soil stratum are composed of the linear hysteretic type viscoelastic media, and their generalized Lame's constants are expressed by

$$\lambda = \lambda_0(1+iD)$$
$$\mu = \mu_0(1+iD)$$

In the numerical integration and summation to obtain the frequency responses, there are no singular points such as poles and branch points as long as the real-valued wave number parameters are concerned, because of the presence of dissipative damping in the media (I) and (II). Therefore ordinary methods of computation can be applied while appropriate interpolation technique is necessary in evaluating multiple integrals.

In the similar dimensionless way as mentioned previously, all elements of the simultaneous equations derived from Eq.(11) can be presented, as shown in the following appendix I.

By using the solutions of the simultaneous equations, the frequency responses of these horizontal translational systems are obtained, and as an example of their responses, the displacements at the origin-point of the coordinate system (r=θ=z=0) subjected to the horizontal force on the head of foundation which are derived from Eq.(12), are shown as;

$$u_0^{(H)} = \begin{bmatrix}u\\ \vartheta\end{bmatrix}^{(H)}_{r=\theta=z=0} = u_0^i + \sum_n a_n^0 Y_n + \int d\zeta \sum_n b_{\zeta n}^0 Y_n \qquad (15)$$

where for the responses subjected to the bed rock motion, the similar representation is obtained except for the first term in the right hand side of this equation, and the coefficient functions of this form are shown in the appendix II.

ROTATIONAL EXCITATION ABOUT A HORIZONTAL AXIS

For the case of the rotational motion subjected to the concentrated rotating force on the head of foundation, the incident-field of the medium (I) expanded in the circumferential direction as shown in Eq.(7) is obtained in the following closed and triangular forms for the radial direction;

$$\begin{bmatrix}U\\ W\end{bmatrix}^{i(R)}_{I} = \frac{\tau_R}{\mu a k^2 \cos jH}\begin{bmatrix}U_z^i\\ j\sin j(H-z)\end{bmatrix}_I \qquad (16)$$

$$U_z^i = \cos j(H-z) - \cos kz\cdot \sec kH + 2jk^{-1}\sin jH\cdot \sec kH\cdot \sin k(H-z)$$

and for the medium (II), any component of the incident-field is evaluated to be zero, as similarly shown for the case of the horizontal translation subjected to the concentrated force on the head of foundation. In this field τ_R is the maximum amplitude of the rotatingly distributed force on the head of foundation Γ_{I0} and the horizontal component of the incident displacement field is necessary with the verical one.

The interacted-field of the rotational motion has the potential forms given by Eqs.(10-2) and (10-3), and any unknown coefficient of the potentials does not vanish to zero, under the boundary conditions dominant in this rotational case.

Among the kernel functions in the simultaneous equations to determine the unknown coefficients of the potentials derived from Eq.(11), the components $(B^{M1}\ B^{M2})_{n\zeta}$ and $C^M_{m\zeta}$ corresponding to the potential $\Psi^{(r)}$ of the subproblem (r) through that of the subproblem (z) have the terms without the Rayleigh functions propagating to the radial direction, whereas the other components contain the propagating terms, as shown later. Similarly as the horizontal translational cases, the dimensionless parameters with superscript (̄) are introduced, and the dimensionless coefficients of the potentials, the dimensionless displacement field subjected to the rotational excitation are presented by;

$$\begin{bmatrix}\bar{u}\\ \bar{\vartheta}\end{bmatrix}^{(R)} = \frac{\mu_0 a}{\tau_R H^2}\begin{bmatrix}1 & \\ & H\end{bmatrix}\begin{bmatrix}u\\ \partial w/\partial r\end{bmatrix}^{(R)} \qquad (17)$$

$$\begin{bmatrix}\bar{A}_n\\ \bar{B}_n\\ \bar{C}_n\\ \bar{D}_\zeta\\ \bar{E}_\zeta\\ \bar{F}_\zeta\end{bmatrix}^{\nu} = \frac{\mu_0 a}{\tau_R H^2}\begin{bmatrix}1 &&&&& \\ & 1 &&&& \\ && H^{-1} &&& \\ &&& 1 && \\ &&&& 1 & \\ &&&&& H^{-1}\end{bmatrix}\begin{bmatrix}A_n\\ B_n\\ C_n\\ D_\zeta\\ E_\zeta\\ F_\zeta\end{bmatrix}^{\nu}$$

where the elements of the simultaneous equations derived from Eq.(11) are shown in the following appendix III.
And by using the solutions of these simultaneous equations, the frequency responses subjected to the rotating force can be obtained, as mentioned similarly in the case of the horizontal translations.

COUPLED HORIZONTAL TRANSLATION AND ROTATION

In order to construct the model of structure-foundation-ground systems and to obtain the dynamic responses of the foundation, the degrees of freedom of the head of foundation are to be at least two in translational and rotational directions. For instance, in the case of the foundation-ground systems subjected to both concentrated external excitations at the head of foundation and uniformly distributed bed rock motions in the horizontal direction, the stiffness matrix associated with the horizontal and rotational displacement, u_o and ϑ_o at the head of foundation, and the displacement transfer vector for the bed rock motion are expressed in frequency domain as follows;

$$\begin{bmatrix}Q\\ M\end{bmatrix} = \begin{bmatrix}K^{HH} & K^{HR}\\ K^{RH} & K^{RR}\end{bmatrix}\begin{bmatrix}u_o\\ \vartheta_o\end{bmatrix}$$

$$\begin{bmatrix}u_o\\ \vartheta_o\end{bmatrix} = \begin{bmatrix}S^H\\ S^R\end{bmatrix} u_G \qquad (18)$$

where Q and M are the horizontal shearing force and the bending moment at the head, K^{IJ} and S^I are the stiffnesses and amplification functions of the system, respectively, and K^{HR} is equal to K^{RH} in consequence of reciprocity theorem. One of the elements K^{HH}, for example, of the dimensionless stiffness matrix of the foundation-ground system can be analytically determined by using the previously mentioned quantities of Eq.(15). The complete stiffness matrix is constructed by adding the relevant terms to the above basic elements to satisfy the elastic support condition for the rotation at the lower end of foundation.
It can be shown that the rectifying term to satisfy the elastic support condition of the lower end of foundation is determined through the similar procedure to that in this analysis.

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APPENDIX I

Elements of the simultaneous equations for the case of horizontal translation.

(a) Homogeneous coefficients:

$$A_n^{I1} = \begin{bmatrix}\alpha_n I_{2\alpha} & p_n\beta_n I_{2\beta}\\ \alpha_n I_{0\alpha} & p_n\beta_n I_{0\beta}\\ -p_n I_{1\alpha} & -\beta_n^2 I_{1\beta}\end{bmatrix}^{I}$$

$$A_n^{II1}=\begin{bmatrix} \alpha_n K_{2\alpha} & p_n\beta_n K_{2\beta} \\ \alpha_n K_{0\alpha} & p_n\beta_n K_{0\beta} \\ p_n K_{1\alpha} & \beta_n^2 K_{1\beta} \end{bmatrix}^{II} \qquad A_n^{M1}=\begin{bmatrix} -\beta_{nI} I_{2\beta} & \beta_{nII} K_{2\beta} \\ \beta_{nI} I_{0\beta} & \beta_{nII} K_{0\beta} \\ 0 & 0 \end{bmatrix}$$

$$A_n^{I2}=\mu\begin{bmatrix} (2p_n^2-k^2) I_{1\alpha} & 2p_n\beta_n^2 I_{1\beta} \\ -2a^{-1}\alpha_n I_{2\alpha} & -2a^{-1} p_n\beta_n I_{2\beta} \\ -2p_n\alpha_n(I_{0\alpha}+I_{2\alpha}) & -\beta_n(2p_n^2-k^2)(I_{0\beta}+I_{2\beta}) \end{bmatrix}^{I}$$

$$A_n^{II2}=-\mu\begin{bmatrix} (2p_n^2-k^2) K_{1\alpha} & 2p_n\beta_n^2 K_{1\beta} \\ 2a^{-1}\alpha_n K_{2\alpha} & 2a^{-1} p_n\beta_n K_{2\beta} \\ 2p_n\alpha_n(K_{0\alpha}+K_{2\alpha}) & \beta_n(2p_n^2-k^2)(K_{0\beta}+K_{2\beta}) \end{bmatrix}^{II}$$

$$A_n^{M2}=\begin{bmatrix} \beta_{nI} I_{1\beta} & -\beta_{nII} K_{1\beta} \\ 2a^{-1} I_{2\beta}-\beta_{nI} I_{1\beta} & 2a^{-1} K_{2\beta}+\beta_{nII} K_{1\beta} \\ p_n(I_{2\beta}-I_{0\beta}) & p_n(K_{2\beta}-K_{0\beta}) \end{bmatrix}\begin{bmatrix} \mu_I\beta_{nI} & \\ & \mu_{II}\beta_{nII} \end{bmatrix}$$

(b) Elements of kernel functions:

$$B_{n\zeta}^{\nu1}=\frac{(-1)^\nu}{8R_\zeta\cosh\beta}\begin{bmatrix} -\zeta J_2 & & \\ & \zeta J_0 & \\ & & J_1 \end{bmatrix}\begin{bmatrix} 1 & p_n \\ 1 & p_n \\ -p_n & -\beta_n^2 \end{bmatrix}\begin{bmatrix} \beta\cosh\beta & \\ & 1 \end{bmatrix}\begin{bmatrix} G_{\alpha\beta}^{11} & G_{\alpha\beta}^{12} \\ G_{\beta\alpha}^{21} & G_{\beta\alpha}^{22} \end{bmatrix}_{n\zeta}^{\nu}$$

$$B_{n\zeta}^{\nu2}=\frac{\mu_\nu(-1)^\nu}{8R_\zeta\cosh\beta}\begin{bmatrix} -J_1 & & \\ & 2a^{-1}\zeta J_2 & \\ & & \zeta(J_0-J_2) \end{bmatrix}\begin{bmatrix} -(2p_n^2-k^2) & -2p_n\beta_n^2 \\ 1 & p_n \\ -2p_n & -(2p_n^2-k^2) \end{bmatrix}\begin{bmatrix} \beta\cosh\beta & \\ & 1 \end{bmatrix}\begin{bmatrix} G_{\alpha\beta}^{11} & G_{\alpha\beta}^{12} \\ G_{\beta\alpha}^{21} & G_{\beta\alpha}^{22} \end{bmatrix}_{n\zeta}^{\nu}$$

$$C_{m\zeta}^{\nu}=\begin{bmatrix} F_{\beta\alpha}^{11} & F_{\beta}^{12} \\ F_{\beta\alpha}^{21} & F_{\beta}^{22} \end{bmatrix}_{m\zeta}\begin{bmatrix} 1 & \\ & 2\beta_m^2 \end{bmatrix}\begin{bmatrix} \mathfrak{L}_{1\alpha} & \\ & \mathfrak{L}_{1\beta} \end{bmatrix}_{m\zeta}^{\nu} \qquad B_{n\zeta}^{M1}=B_{n\zeta}^{M2}=C_{m\zeta}^{M}=0 \qquad :\nu=I,II$$

$$R_\zeta=-4\alpha\beta\zeta^2(2\zeta^2-k^2)+\alpha\beta[(2\zeta^2-k^2)^2+4\zeta^4]\cosh\alpha\cosh\beta$$
$$-\zeta^2[(2\zeta^2-k^2)^2+4\alpha^2\beta^2]\sinh\alpha\sinh\beta$$

$$G_{\alpha\beta}^{11}=-2\alpha\beta(2\zeta^2-k^2)\alpha^{42}-(2\zeta^2-k^2)^2\sinh\beta\cdot\alpha^{12}+4\alpha\beta\zeta^2\cosh\beta\cdot\alpha^{22}$$

$$G_{\alpha\beta}^{12}=2\zeta^2(2\zeta^2-k^2)\alpha^{32}+(2\zeta^2-k^2)^2\cosh\beta\cdot\alpha^{12}-4\alpha\beta\zeta^2\sinh\beta\cdot\alpha^{22}$$

$$G_{\alpha\beta}^{21}=2\alpha\beta(2\zeta^2-k^2)\alpha^{11}-2\alpha\beta[2\alpha\beta\sinh\beta-(2\zeta^2-k^2)\sinh\alpha]\alpha^{41}$$
$$-[4\alpha\beta\zeta^2\cosh\alpha\cosh\beta-(2\zeta^2-k^2)^2\sinh\alpha\sinh\beta]\alpha^{11}$$

$$G_{\alpha\beta}^{22}=2\alpha\beta[2\zeta^2\cosh\beta-(2\zeta^2-k^2)\cosh\alpha]\alpha^{41}$$
$$+[4\alpha\beta\zeta^2\sinh\alpha\cosh\beta-(2\zeta^2-k^2)^2\cosh\alpha\sinh\beta]\alpha^{11}$$

$$\begin{bmatrix} F^{11}_{\alpha\beta} & F^{12}_{\alpha} \\ F^{21}_{\alpha\beta} & F^{22}_{\alpha} \end{bmatrix}_{n\zeta} = \begin{bmatrix} (2\beta_n^2+k^2)\cosh\alpha & p_n\cosh\alpha \\ (2\beta_n^2+k^2)\sinh\alpha+2(-1)^n p_n\alpha & p_n\sinh\alpha+(-1)^n\alpha \end{bmatrix}$$

$$\begin{bmatrix} \alpha^{11} & \alpha^{12} \\ \alpha^{22} & \alpha^{21} \\ \alpha^{32} & \\ \alpha^{41} & \alpha^{42} \end{bmatrix}_{n\zeta} = \frac{2}{(\alpha^2+p_n^2)} \begin{bmatrix} (-1)^{n+1}\alpha\cosh\alpha & (-1)^{n+1}p_n\sinh\alpha-\alpha \\ (-1)^{n+1}p_n\cosh\alpha & (-1)^{n+1}\alpha\sinh\alpha+p_n \\ & \alpha\cosh\alpha \\ p_n\cosh\alpha & (-1)^{n+1}p_n+\alpha\sinh\alpha \end{bmatrix}$$

$$\begin{bmatrix} \mathfrak{L}^{I}_{1\alpha} \\ \mathfrak{L}^{II}_{1\alpha} \end{bmatrix} = \frac{\zeta a}{\zeta^2+\alpha_n^2} \begin{bmatrix} -J_0 I_{1\alpha}+\alpha_{nI} J_1 I_{0\alpha} \\ J_0 K_{1\alpha}+\alpha_{nII} J_1 K_{0\alpha} \end{bmatrix} \qquad \begin{bmatrix} J_\ell \\ I_{\ell\alpha} \\ K_{\ell\alpha} \end{bmatrix} = \begin{bmatrix} J_\ell(\zeta a) \\ I_\ell(\alpha_{nI} a) \\ K_\ell(\alpha_{nII} a) \end{bmatrix}$$

(c) Inhomogeneous terms:

$$\begin{bmatrix} f^1 \\ f^2 \end{bmatrix}^{(H)}_n = - \begin{bmatrix} 0 \\ 2U^c \\ 0 \\ 0 \\ 0 \\ 2\mu T^s_{rz} \end{bmatrix}^{i(H)}_I \qquad \begin{bmatrix} f^1 \\ f^2 \end{bmatrix}^{(G)}_n = \sum_{\nu=I}^{II} (-1)^\nu \begin{bmatrix} 0 \\ 2U^c \\ 0 \\ 0 \\ 0 \\ 2\mu T^s_{rz} \end{bmatrix}^{i(G)}_\nu$$

$$\begin{bmatrix} U^c \\ T^s_{rz} \end{bmatrix}^{i(H)}_I = \frac{2}{\mu\beta_n^2} \begin{bmatrix} 1 \\ -p_n \end{bmatrix}_I \qquad \begin{bmatrix} U^c \\ T^s_{rz} \end{bmatrix}^{i(G)}_\nu = \frac{2(-1)^{n+1}}{\beta_n^2} \begin{bmatrix} p_n \\ -k^2 \end{bmatrix}_\nu$$

APPENDIX II

Elements of the response function subjected to the horizontal excitation at the head of foundation, presented by Eq.(15).

$$u^i_o = \frac{\tan k}{\mu k} \begin{bmatrix} 1 \\ 0 \end{bmatrix}^I \qquad a^0_n = \frac{1}{2} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} \alpha_n & \beta_n & p_n\beta_n \end{bmatrix}^I$$

$$b^0_{\zeta n} = \frac{\zeta}{16R_\zeta\cosh\beta} \begin{bmatrix} 2(\cosh\alpha-\cosh\beta) & (2\zeta^2-k^2)\sinh\alpha-2\alpha\beta\sinh\beta \\ 0 & \alpha k^2\cosh\beta \end{bmatrix} *$$

$$* \begin{bmatrix} \alpha\beta[2\zeta^2\cosh\beta-(2\zeta^2-k^2)\cosh\alpha] & -\zeta^2[2\alpha\beta\sinh\beta-(2\zeta^2-k^2)\sinh\alpha] \\ 2\alpha\beta\sinh\alpha-(2\zeta^2-k^2)\sinh\beta & -[2\zeta^2\cosh\alpha-(2\zeta^2-k^2)\cosh\beta] \end{bmatrix} *$$

$$* \begin{bmatrix} F^{11}_{\beta\alpha} & 0 & 2\beta_n^2 F^{12}_{\beta} \\ F^{21}_{\beta\alpha} & 0 & 2\beta_n^2 F^{22}_{\beta} \end{bmatrix} \begin{bmatrix} \mathfrak{L}_{1\alpha} & & \\ & 0 & \\ & & \mathfrak{L}_{1\beta} \end{bmatrix}^I_{n\zeta}$$

APPENDIX III

Elements of the simultaneous equations for the case of rotational motion.

(a) Homogeneous coefficients:
When the homogeneous coefficients for the case of horizontal translation is $A_n^{(H)}(p_n)$, presented in the appendix I, those for the case of rotation $A_n^{(R)}(p_n)$ is obtained as follows;

$$A_n^{(R)}(p_n) = A_n^{(H)}(-p_n)$$

(b) Elements of kernel functions:

$$B_{n\zeta}^{\nu 1} = \frac{(-1)^{\nu+1}}{8R_\zeta \alpha\cosh\alpha} \begin{bmatrix} -\zeta J_2 & & \\ & \zeta J_0 & \\ & & J_1 \end{bmatrix} \begin{bmatrix} 1 & -p_n \\ 1 & -p_n \\ p_n & -\beta_n^2 \end{bmatrix} \begin{bmatrix} \zeta^2 & \\ & \alpha\cosh\alpha \end{bmatrix} \begin{bmatrix} G_{\alpha\beta}^{22} & G_{\alpha\beta}^{21} \\ G_{\beta\alpha}^{12} & G_{\beta\alpha}^{11} \end{bmatrix}_{n\zeta}^{\nu}$$

$$B_{n\zeta}^{\nu 1} = \frac{(-1)^{\nu}\mu_\nu}{8R_\zeta \alpha\cosh\alpha} \begin{bmatrix} -J_1 & & \\ & 2a^{-1}\zeta J_2 & \\ & & \zeta(J_0-J_2) \end{bmatrix} \begin{bmatrix} -(2p_n^2-k^2) & -2p_n\beta_n^2 \\ 1 & p_n \\ -2p_n & -(2p_n^2-k^2) \end{bmatrix} \begin{bmatrix} \zeta^2 & \\ & \alpha\cosh\alpha \end{bmatrix} \begin{bmatrix} G_{\alpha\beta}^{22} & G_{\alpha\beta}^{21} \\ G_{\beta\alpha}^{12} & G_{\beta\alpha}^{11} \end{bmatrix}_{n\zeta}^{\nu}$$

$$B_{n\zeta}^{Mj} = (D^{Ij} \quad D^{IIj})_{n\zeta} \qquad :j=1,2$$

$$D_{n\zeta}^{\nu 1} = \frac{(-1)^{\nu+1}}{\beta\cosh\beta} \begin{bmatrix} -\zeta J_2 & & \\ & \zeta J_0 & \\ & & J_1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} \beta^{21} \\ \beta^{11} \end{bmatrix}_{n\zeta}^{\nu\, T}$$

$$D_{n\zeta}^{\nu 2} = \frac{(-1)^{\nu+1}\mu_\nu}{\beta\cosh\beta} \left(-\begin{bmatrix} -\zeta J_2 & & \\ & \zeta J_0 & \\ & & J_1 \end{bmatrix} \begin{bmatrix} \beta_n^2 \\ 1 \\ p_n \end{bmatrix} + \begin{bmatrix} 0 & & \\ & J_1 & \\ & & \zeta J_0 \end{bmatrix} \begin{bmatrix} 0 \\ -\beta_n^2 \\ 2p_n \end{bmatrix} \right) \begin{bmatrix} \beta^{21} \\ \beta^{11} \end{bmatrix}_{n\zeta}^{\nu\, T}$$

$$C_{m\zeta}^{\nu} = \begin{bmatrix} F_{\alpha}^{22} & F_{\alpha\beta}^{21} \\ F_{\alpha}^{12} & F_{\alpha\beta}^{11} \end{bmatrix}_{m\zeta} \begin{bmatrix} 2 & \\ & -1 \end{bmatrix} \begin{bmatrix} \tilde{L}_{1\alpha} & \\ & \tilde{L}_{1\beta} \end{bmatrix}_{m\zeta}^{\nu}$$

$$C_{m\zeta}^{M} = \begin{bmatrix} E^{I} & o \\ o & E^{II} \end{bmatrix}_{m\zeta}$$

$$E_{m\zeta}^{\nu} = \begin{bmatrix} p_m \sinh\beta + (-1)^m \beta \\ -p_m \cosh\beta \end{bmatrix}^{\nu} \tilde{L}_{1\beta}^{\nu} \qquad :\nu=I,II$$

where $G_{\alpha\beta}^{ij}$, $F_{\alpha\beta}^{i1}$, F_{α}^{i2} $(i,j=1,2)$, $\tilde{L}_{1\beta}^{\nu}$ $(\nu=I,II)$ and J_ℓ $(\ell=0,1,2)$ are shown in the appendix I.

(c) Inhomogeneous terms:

$$\begin{bmatrix} f^1 \\ f^2 \end{bmatrix}_n^{(R)} = - \begin{bmatrix} 0 \\ 2U^s \\ rW^c \\ r\mu T^s_{rr} \\ 0 \\ 2\mu T^c_{rz} \end{bmatrix}_I^{i(R)}$$

$$\begin{bmatrix} U^s \\ W^c \\ T^s_{rr} \\ T^c_{rz} \end{bmatrix}_I^{i(R)} = \frac{2}{\mu a k^2 \alpha_n^2} \begin{bmatrix} p_n \\ j^2 \\ -p_n j^2 \\ 2j^2 \end{bmatrix}_I + \frac{2}{\mu a k^3 \beta_n^2 \cos j \cdot \cos k} \begin{bmatrix} k \sin p_n (k \sin k - 2j \sin j) \\ + j p_n (2 \sin j \cdot \sin k - 1) \\ 0 \\ 0 \\ -k p_n \sin p_n (k \sin k + 2j \sin j) \\ + k^2 (2j \sin j \cdot \sin k + k) \end{bmatrix}_I$$